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Fast Radio Bursts from Axion Stars

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Axions are one of the most promising candidates of dark matter. The axions have been shown to form miniclusters with masses $\sim 10^{-12}M_\odot$ and to become dominant component of dark matter. Some of the axion miniclusters condense to form axion stars. We have recently shown a possible origin of fast radio bursts (FRBs) by assuming that the axion stars are main component of halos: FRBs arise from the collisions between the axion stars and neutron stars. It is remarkable that the masses of the axion stars obtained by the comparison of the theoretical and observational event rates are coincident with the mass $\sim 10^{-12}M_\odot$. In this paper, we describe our model of FRBs in detail. We derive the approximate solutions of the axion stars with large radii and constraint their masses for the approximation to be valid. The FRBs are emitted by electrons in atmospheres of neutron stars. By calculating the optical depth of the atmospheres, we show that they are transparent for the radiations with the frequency given by the axion mass m_a such as $m_a/2\pi \simeq 2.4\text{GHz}(m_a/10^{-5}\text{eV})$. Although the radiations are linearly polarized when they are emitted, they are shown to be circularly polarized after they pass magnetospheres of neutron stars. We also show that the FRBs are not broadband and their frequencies have finite bandwidths owing to the thermal fluctuations of the electrons in the atmospheres of the neutron stars. The presence of the finite bandwidths is a distinctive feature of our model and can be tested observationally. Furthermore, we show that similar FRBs may arise when the axion stars collide with magnetic white dwarfs $B \sim 10^9\text{G}$. The distinctive feature is that the durations of the bursts are of the order of 0.1second and that the radiations have wider bandwidths than those of the radiations produced by the collisions with neutron stars.

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I. INTRODUCTION

Fast Radio Bursts have recently been discovered[1–3] at around 1.4 GHz frequency. The durations of the bursts are typically a few milliseconds. The origin of the bursts has been suggested to be extra-galactic owing to their large dispersion measures. This suggests that the large amount of the energies $\sim 10^{43}\text{GeV/s}$ is produced at the radio frequencies. The event rate of the burst is estimated to be $\sim 10^{-3}$ per year in a galaxy. Furthermore, no gamma or X ray bursts associated with the bursts have been detected. Follow up observations[4] of FRBs do not find any signals from the direction of the FRB. To find progenitors of the bursts, several models[5] have been proposed. They ascribe FRBs to traditional sources such as neutron star-neutron star mergers, magnetars, black holes, et al..

Our model[6, 7] ascribes FRBs to axions[8], which are one of most promising candidates of dark matter. A prominent feature of axions is that they are converted to radiations under strong magnetic fields. The axions form axion stars known as oscillaton[9] made of axions bounded gravitationally. The axion stars are condensed objects of axion miniclusters[10], which have been shown to be produced after the QCD phase transition and to form the dominant component of dark matter in the Universe. Furthermore, the axion miniclusters have been shown to form the axion stars by gravitationally losing their kinetic energies[10, 11]. Thus, the axion stars are the dominant component of dark matter. We have recently proposed a progenitor of the FRBs that the FRBs arise from the collisions between the axion stars and neutron stars. All of the properties (duration, event rate and total radiation energy) of the FRBs observed can be naturally explained in our model.

In the paper, we present the details of our models. First of all, we derive the solutions of the axion stars by expanding the axion field and the gravitational fields in terms of eigen modes $\cos(n\omega t)$ with n integers, where the eigen value ω can be determined by solving axion field equation representing gravitationally bounded axions. It is given such that $\omega = m_a - k^2/2m_a$ where $k^2/2m_a (\ll m_a)$ denotes a gravitational binding energy of an axion bounded by the axion star; $k \simeq GM_a m_a^2$ where G and M_a are the gravitational constant and the mass of the axion star, respectively. Then, we find that the mass M_a of the axion star is given in terms of the radius $R_a = 1/k$ of the axion stars such that $M_a = 1/(Gm_a^2 R_a)$. The value of the mass M_a can be obtained by the comparison of the theoretical and observational even rate of FRBs in our production mechanism of the FRBs. It is remarkable that a mass $M_a \sim 10^{-12}M_\odot$ obtained

in such a way is coincident with the mass of the axion miniclusters previously obtained. The radius R_a is given by $R_a \sim 10^2 \text{km}$, which is much larger than neutron stars.

The field configurations of the solutions are not static but oscillating and localized with radius $R_a = 1/k$; $a(t, \vec{x}) \propto \cos(\omega t) \exp(-k|\vec{x}|) \simeq \cos(m_a t) \exp(-k|\vec{x}|)$. We show that the oscillating electric fields $\vec{E}_a(t, \vec{x}) \propto a(t, \vec{x}) \vec{B}(\vec{x})$ are generated on the axion stars under external magnetic fields $\vec{B}(\vec{x})$. Thus, when the axion stars collide with neutron stars with strong magnetic fields, the electric fields $\vec{E}_a(t, \vec{x})$ are generated, which make electrons in atmospheres[12] of neutron stars coherently oscillate. Thus, the electrons emit coherent radiations with the frequency given by the axion mass. Since the electrons are much dense in the atmospheres, the large amount of radiations with the frequency $m_a/2\pi \simeq 2.4 \text{GHz}$ ($m_a/10^{-5} \text{eV}$) can be produced in the collisions. The total amount of the energy of the radiations is given by $10^{-12} M_\odot (10 \text{km}/10^2 \text{km})^2 \sim 10^{43} \text{GeV}$, where the radii of the neutron stars and the axion stars are supposed to be 10km and 10^2km , respectively. This is our production mechanism of FRBs.

We also discuss the optical properties of the atmospheres of neutron stars. The geometrical depth of the hydrogen atmospheres of old neutron stars is of the order of 0.1cm . The radiations emitted in the atmospheres can pass through them because they are shown to be optically thin for the radiations with the frequency ($m_a/2\pi$) $\simeq 2.4 \text{GHz}$ ($m_a/10^{-5} \text{eV}$). They can also pass through magnetospheres of neutron stars. After they pass the magnetospheres, the radiations are circularly polarized owing to the absorption of the radiations with either right or left handed polarization. The circular polarization of a FRB has recently been observed[4].

Although the radiations produced by our mechanism are monochromatic having the frequency given by $\omega = m_a/2\pi$, the observed radiations have bandwidths at least wider than the range $1.2 \text{GHz} \sim 1.6 \text{GHz}$ used by actual observations. We show that the bandwidths ω_{th} ($\omega \pm \omega_{\text{th}}$) of FRBs arise from thermal fluctuations of electrons. Thus, the bandwidths are narrow. The presence of such narrow bandwidths owing to the thermal fluctuations is a distinctive feature of our model and can be tested observationally.

The relative velocities v_c in the collisions are given by $\sqrt{2GM_{ns}/R_{ns}}$, which is of the order of 10^5km/s ; M_{ns} (R_{ns}) denote mass (radius) of neutron stars. The fact explains the duration of the FRBs being of the order of $\sim 1 \text{ms}$. The radiations observed at the rest frame of neutron stars are affected by Doppler effect. Furthermore, the radiations observed at the earth are affected by gravitational ($\sqrt{1 - 2GM_{ns}/R_{ns}}$) and cosmological redshifts z . Owing to the effects, the actual frequencies observed at the earth are given by $(m_a/2\pi) \times (1 - 2GM_{ns}/R_{ns})/(1 + z)$ which is less than $(m_a/2\pi) \simeq 2.4 \text{GHz}$ ($m_a/10^{-5} \text{eV}$).

It is interesting to see that similar radio bursts are emitted when the axion stars collide with white dwarfs with very strong magnetic fields $\sim 10^9 \text{G}$. In particular, the duration of the bursts is of the order of 0.1second , contrary to the observed FRBs. This is because the relative velocities between the axion stars and the white dwarfs in the collisions are given by $\sqrt{2GM_{wd}/R_{wd}} \sim 4 \times 10^3 \text{km/s}$, where $M_{wd} = 0.5 M_\odot$ and $R_{wd} = 10^4 \text{km}$ denote the typical mass and radius of the white dwarfs, respectively. Furthermore, the bandwidths of the radiations are wider than those of radiations emitted by neutron stars. This is because the magnetic fields of the white dwarfs are much weaker than those of neutron stars. Thus, we can distinguish them from the radiations emitted from neutron stars. The production rate of the bursts is larger than the one of the FRBs observed, if the number of the white dwarfs with strong magnetic fields $B \geq 10^9 \text{G}$ is larger than 10^7 in a galaxy. Furthermore, their luminosities are much larger than those of the FRBs observed. On the other hand, the number of typical white dwarfs with magnetic fields $\sim 10^7 \text{G}$ is much larger than the one of the white dwarfs with $B \geq 10^9 \text{G}$. However, it is difficult to observe the radiations from the white dwarfs with small magnetic fields $\leq 10^7 \text{G}$ because the total amount of the radiation energies is not sufficiently large to be observable.

In the next section (II), we derive approximate solutions of axion stars with small masses coupled with gravity. We can see that the axion field oscillates with the frequency $m_a/2\pi$. In the section (III), we show that electric fields are produced on axion stars under external magnetic fields. They are parallel to the magnetic fields and oscillate with frequency $m_a/2\pi$. In the section (IV), we determine masses of axion stars by comparison of theoretical with observed rates of FRBs. The masses are found to be coincident with those estimated previously as the masses of the axion miniclusters. In the section (V), we describe how the radiations are emitted from the collisions. Especially, we show that they are emitted from atmospheres of neutron stars. We find that once the axion stars touch the atmospheres, they lose their energies by emitting radiations. In the section (VI), we show by calculating optical depth that the atmospheres are transparent for the radiations. We also discuss that the radiations are circularly polarized after they pass magnetospheres of neutron stars. In the section (VII), we discuss how the thermal fluctuations of electrons emitting FRBs give rise to narrow bandwidths of the radiations. In the section (VIII), we discuss that similar RFBs arise in the collisions between axion stars and magnetic white dwarfs. We find that their durations are of the order of 0.1 second and their frequencies have bandwidth wider than those of FRBs from neutron stars. We summarize our results in the final section (IX).

II. AXION STARS

First we would like to make a brief review of axions and axion stars. Axions described by a real scalar field a are Nambu-Goldstone boson associated with Pecci-Quinn global U(1) symmetry[13]. The symmetry was introduced to cure strong CP problems in QCD. After the breakdown of the Pecci-Quinn symmetry at the period of much higher temperature than 1GeV, axions are thermally produced as massless particles in the early Universe. They are however only minor components of dark matter. Since the axions interact with instanton density in QCD, the potential term $-f_a^2 m_a^2 \cos(a/f_a)$ develops owing to instanton effects at the temperature below 1GeV; f_a denotes the decay constant of the axions. Thus, the axion field oscillates around the minimum $a = 0$ of the potential. But, the initial value of the field a at the temperature 1GeV is unknown. It can take a different value in a region from those in other regions causally disconnected. Thus, there are many regions causally disconnected at the epoch around the temperature 1GeV, in each of which the axion field takes a different initial value; energy density is also different. With the expansion of the Universe, the regions with different energy densities are causally connected. Thus, there arises spatial fluctuations of the axion energy density. The nonlinear effects of the axion potential cause the fluctuations with over densities in some regions grow to form axion miniclusters[10] at the period of equal axion-matter radiation energy density in the regions. Their masses have been estimated to be of the order of $10^{-12} M_\odot$. Furthermore, these miniclusters condense to form axion stars with gravitationally losing their kinetic energies[11]. Therefore, masses of axion stars are expected to be of the order of $10^{-12} M_\odot$.

Now we explain the classical solutions of the axion stars obtained in previous papers[11, 14–16]. The solutions are found by solving classical equations of axion field $a(\vec{x}, t)$ coupled with gravity. In particular we would like to obtain spherical symmetric solutions of axion stars with much smaller masses than the critical mass[16] M_{\max} of the axion stars; axion stars are stable when their masses are smaller than the critical mass $M_{\max} \sim 0.6 m_{\text{pl}}^2/m_a \simeq 0.5 \times 10^{-5} M_\odot (10^{-5} \text{eV}/m_a)$. The mass is obtained only for free axion field without self-interaction. The gravity of the axion stars with much small masses is much weak so that we may take the space-time metrics given by

$$ds^2 = (1 + h_t)dt^2 - (1 + h_r)dr^2 - r^2(d\theta^2 + \sin\theta d\phi^2) \quad (1)$$

with both h_t and $h_r \ll 1$. It is easy to derive the equations of motion of the axion field and gravity,

$$\begin{aligned} (1 - h_t)\partial_0^2 a &= \frac{\partial_0 h_t - \partial_0 h_r}{2} \partial_0 a + (1 - h_r)(\partial_r^2 + \frac{2}{r}\partial_r)a + \frac{\partial_r h_t - \partial_r h_r}{2} \partial_r a - m_a^2 a \\ \partial_r h_t &= \frac{h_r}{r} + 4\pi G r \left((\partial_r a)^2 - m_a^2 a^2 + (\partial_0 a)^2 \right) \\ \partial_r h_r &= -\frac{h_r}{r} + 4\pi G r \left((\partial_r a)^2 + m_a^2 a^2 + (\partial_0 a)^2 \right) \end{aligned} \quad (2)$$

with the gravitational constant G , where we assume the axion potential such that $V_a = -f_a^2 m_a^2 \cos(a/f_a) \simeq -f_a^2 m_a^2 + m_a^2 a^2/2$ for $a/f_a \ll 1$. As we will see later, the assumption holds for the axion stars with small masses, e.g. $10^{-12} M_\odot$.

It is well known that there are no static solutions of real scalar fields coupled with gravity. The fields and the metrics oscillate with time. We expand the axion field and the metric such that

$$\begin{aligned} a(t, r) &= \sum_{n=0,1,2,\dots} a_n(r) \cos((2n+1)\omega t) = a(r) \cos(\omega t) + a_1(r) \cos(3\omega t), \dots, \\ h_{t,r}(t, r) &= \sum_{n=0,1,2,\dots} h_{t,r}^n(r) \cos(2n\omega t) = h_{t,r}^0(r) + h_{t,r}^1(r) \cos(2\omega t), \dots, \end{aligned} \quad (3)$$

with $a(r) \equiv a_0(r)$. Then, only by taking the terms proportional to $\cos^0(\omega t) = 1$ and $\cos(\omega t)$ in the equations (2), we obtain

$$-\omega^2(1 - h_t^0 + \frac{h_r^1 - h_t^1}{2})a(r) = -\omega^2 \frac{h_r^1 - h_t^1}{2} a(r) + (1 - h_r^0)(\partial_r^2 + \frac{2\partial_r}{r})a(r) - m_a^2 a(r) \quad (4)$$

$$\begin{aligned} &-\frac{1}{2}(\partial_r h_r^0 - \partial_r h_t^0 + \frac{\partial_r h_r^1 - \partial_r h_t^1}{2})\partial_r a(r) \\ \partial_r h_t^0 &= \frac{h_r^0}{r} + 2\pi G r \left((\partial_r a(r))^2 - m_a^2 a^2(r) + \omega^2 a^2(r) \right) \end{aligned} \quad (5)$$

$$\partial_r h_r^0 = -\frac{h_r^0}{r} + 2\pi Gr \left((\partial_r a(r))^2 + m_a^2 a^2(r) + \omega^2 a^2(r) \right) \quad (6)$$

$$\partial_r h_t^1 = \frac{h_r^1}{r} + 2\pi Gr \left((\partial_r a(r))^2 - m_a^2 a^2(r) - \omega^2 a^2(r) \right) \quad (7)$$

$$\partial_r h_r^1 = -\frac{h_r^1}{r} + 2\pi Gr \left((\partial_r a(r))^2 + m_a^2 a^2(r) - \omega^2 a^2(r) \right). \quad (8)$$

We note that when the gravitational effects vanish i.e. $G \rightarrow 0$, there is a solution $a = \tilde{a}_0 \cos(m_a t)$ with $\omega = m_a$ as well as $h_{t,r}^{0,1} = 0$. Since we consider axion stars with small masses, ω is almost equal to m_a ; $m_a^2 - \omega^2 \ll m_a^2$. That is, the binding energies $m_a - \omega$ are much smaller than m_a . Furthermore, as we will see later, the radius R_a of the axion stars with small masses is very large; $R_a \gg m_a^{-1}$. Thus, the term $\partial_r a(r)$ is much smaller than the term $m_a a(r)$, i.e. $(\partial_r a(r))^2 \ll (m_a a(r))^2$. We may approximate the above equations in the following,

$$(m_a^2 - \omega^2)a(r) + m_a^2 h_t^0 a(r) = (\partial_r^2 + \frac{2\partial_r}{r})a(r) \quad (9)$$

$$\partial_r h_t^0 \simeq \frac{h_r^0}{r} \quad (10)$$

$$\partial_r h_r^0 \simeq -\frac{h_r^0}{r} + 4\pi Gr m_a^2 a^2(r) \quad (11)$$

$$\partial_r h_t^1 \simeq \frac{h_r^1}{r} - 4\pi Gr m_a^2 a^2(r) \quad (12)$$

$$(13)$$

with $h_r^1 \ll h_{t,r}^0 \ll 1$ and $h_r^1 \ll h_t^1 \ll 1$, since $h_r^1 \sim O(Gr^2((\partial_r a)^2 + (m_a^2 - \omega^2)a^2))$ and the other metrics $h \sim O(Gr^2 m_a^2 a^2)$.

Therefore, we obtain the equation of the axion field,

$$-\frac{k^2}{2m_a}a(r) = -\frac{1}{2m_a}(\partial_r^2 + \frac{2\partial_r}{r})a(r) + m_a \phi a(r) \quad (14)$$

with $k^2 \equiv m_a^2 - \omega^2$, where “gravitational potential” $\phi \equiv h_t^0/2$ satisfies

$$(\partial_r^2 + \frac{2\partial_r}{r})\phi = 2\pi G m_a^2 a^2(r). \quad (15)$$

Obviously, $k^2/2m_a$ represents a binding energy of the axion bounded to an axion star whose mass M_a is given by $M_a = \int d^3x ((\partial_0 a)^2 + (\partial_r a)^2 + m_a^2 a^2)/2 \simeq \int d^3x m_a^2 a^2(t, r)/2 = \int d^3x m_a^2 a^2(r)/2$ with the average taken in time; $\omega^2 \simeq m_a^2$. The equation (14) can be rewritten in the limit $r \rightarrow \infty$ as

$$-\frac{k^2}{2m_a}a(r) = -\frac{1}{2m_a}(\partial_r^2 + \frac{2\partial_r}{r})a(r) - \frac{Gm_a M_a}{r}a(r). \quad (16)$$

A solution in eq(16) is given by $a(r) = \tilde{a}_0 \exp(-kr)$ with $k = Gm_a^2 M_a$. Thus, we find that the radius $R_a = k^{-1} = (Gm_a^2 M_a)^{-1}$ of the axion star is much larger than m_a^{-1} for small mass M_a . We can confirm numerically that the solution in eq(16) represents approximate solutions of the equations (14) and (15). In this way we approximately obtain spherical symmetric solutions,

$$a(\vec{x}, t) = a_0 f_a \exp(-\frac{r}{R_a}) \cos(m_a t), \quad (17)$$

with $r = |\vec{x}|$. The solutions represent boson stars made of the axions bounded gravitationally, named as axion stars. The solutions are valid for the axion stars with small masses $M_a \ll 10^{-5} M_\odot$. The radius R_a of the axion stars is numerically given in terms of the mass M_a by

$$R_a = \frac{m_{\text{pl}}^2}{m_a^2 M_a} \simeq 260 \text{ km} \left(\frac{10^{-5} \text{ eV}}{m_a} \right)^2 \left(\frac{10^{-12} M_\odot}{M_a} \right), \quad (18)$$

with the Planck mass m_{pl} . The coefficient a_0 can be obtained by using the relations $M_a \simeq \int d^3x m_a^2 a(r)^2 / 2 = \pi m_a^2 a_0^2 f_a^2 R_a^3 / 4$ and $m_a \simeq 6 \times 10^{-6} \text{eV} \times (10^{12} \text{GeV} / f_a)$, where the average is taken in time,

$$a_0 \simeq 0.9 \times 10^{-6} \left(\frac{10^2 \text{km}}{R_a} \right)^2 \frac{10^{-5} \text{eV}}{m_a}. \quad (19)$$

Thus, the condition $a/f_a \ll 1$ is satisfied for the axion stars with small mass $M_a \sim 10^{-12} M_\odot$. We have simply used the mass $10^{-12} M_\odot$ for reference. But, the mass is the one we determine by the comparison of the theoretical and observational event rates of FRBs, as we show below. The mass is much smaller than the critical mass $M_{\text{max}} \sim 10^{-5} M_\odot$. Thus, the solutions represent stable axion stars. In this way the solutions along with the parameters R_a and a_0 can be approximately obtained. Obviously, the axion stars are composed of axions with much small momenta $\sim 1/R_a$.

Here we should make a comment on the quartic term $-(m_a^2/f_a^2)a^4/24$ of the potential $V_a = -f_a^2 m_a^2 \cos(a/f_a) \simeq -f_a^2 m_a^2 + m_a^2 a^2/2 - (m_a^2/f_a^2)a^4/24$. We have neglected the term in the above discussion. Although the term is much smaller than the mass term $m_a^2 a^2/2$, the term gives a contribution $-m_a^2 a(r)^3/(12f_a^2)$ in the eq(9) which is comparable to the term $(\omega^2 - m_a^2)a(r) \simeq R_a^{-2}a(r)$ when $R_a \simeq 130 \text{km}$ or less. Since the quartic term is negative in the potential, the masses of the axion stars decrease with the increase of the field amplitude a_0 . That is, the axion stars become unstable when the term is comparable to the term $(\omega^2 - m_a^2)a(r)$. Thus, our solutions are only stable for the axion stars with larger radii than 130km (smaller masses than $0.2 \times 10^{-11} M_\odot$.) This indicates that the critical mass becomes much smaller than M_{max} when the quartic term is taken into account. We do not yet know real critical mass when we take account of the full potential V_a of the axions. But the axion stars at least with masses smaller than $0.2 \times 10^{-11} M_\odot$ are stable since the approximation of neglecting the quartic term is valid. (Much small critical masses $\sim 10^{-21} M_\odot$ have been previously pointed out[17] using the procedure in the previous work[18]. But the procedure is only valid for free fields coupled with gravity. Especially, it is not applicable for the real scalar fields with nonlinear interactions such as the axions. The approximations such as $\langle \hat{a}^4 \rangle = \langle \hat{a}^2 \rangle^2$ is used in the reference for obtaining the axion stars where \hat{a} represents axion field operator and the state $|\rangle$ does a state with number of axions fixed, i.e. eigenstates of the axion number operator. On the other hand, our classical approximation is to assume that the state $|\rangle$ represents a coherent state of axions. Thus, we have $\langle \hat{a}^4 \rangle = \langle \hat{a} \rangle^4$. The critical mass shown in the previous paper is of the order of $10^{-20} M_\odot$, while as we have shown perturbatively, the critical masses are of the order of $10^{-12} M_\odot$. The difference comes from the use of the different approximations. We point out that the critical masses we obtain are of the same order of the magnitude as the ones shown in the paper[7]. Anyway, more rigorous treatments are needed to see the precise values of the critical mass.)

In our analysis, the radius 130km of the axion stars with the mass $0.2 \times 10^{-11} M_\odot$ was roughly derived only as a guide for a critical radius. We may use the radius 10²km as a reference of the stable axion stars in the discussion below.

III. AXION STARS IN MAGNETIC FIELDS

We proceed to discuss electric field \vec{E}_a generated on the axion stars under magnetic field \vec{B} . It is well-known that the axion couples with both electric \vec{E} and magnetic fields \vec{B} in the following,

$$L_{aEB} = k\alpha \frac{a(\vec{x}, t) \vec{E} \cdot \vec{B}}{f_a \pi} + \frac{\vec{E}^2 - \vec{B}^2}{2} \quad (20)$$

with the fine structure constant $\alpha \simeq 1/137$, where the numerical constant k depends on axion models; typically it is of the order of one. Hereafter we set $k = 1$. From the Lagrangian, we derive the Gauss law, $\vec{\partial} \cdot \vec{E} = -\alpha \vec{\partial} \cdot (a \vec{B}) / f_a \pi$. Thus, the electric field generated on the axion stars under the magnetic field \vec{B} is given by

$$\vec{E}_a(r, t) = -\alpha \frac{a(\vec{x}, t) \vec{B}}{f_a \pi} = -\alpha \frac{a_0 \exp(-r/R_a) \cos(m_a t) \vec{B}(\vec{r})}{\pi} \quad (21)$$

$$\simeq 0.4 \times 10^4 \text{eV} (= 2 \times 10^4 \text{eV/cm}) \cos(m_a t) \left(\frac{10^2 \text{km}}{R_a} \right)^2 \frac{10^{-5} \text{eV}}{m_a} \frac{B}{10^{10} \text{G}}. \quad (22)$$

We find that the electric field is very strong at neutron stars with magnetic fields $\sim 10^{10} \text{G}$, while it is much weak at the sun with magnetic field $\sim 1 \text{G}$. The electric field \vec{E}_a is parallel to the magnetic field \vec{B} and oscillates coherently

over the whole of the axion stars. When the axion stars are in magnetized ionized gases, the field induces coherently oscillating electric currents with large length scale R_a of the axion stars. Thus the large amount of dipole radiations can be emitted. Especially, electrons in the atmospheres of neutron stars oscillate and emit coherent dipole radiations with the frequency $m_a/2\pi$. We should mention that the motions of charged particles accelerated by the electric field \vec{E}_a are not affected by the magnetic field \vec{B} since \vec{B} is parallel to \vec{E} . Thus, they can emit dipole radiations.

Here we make a comment that the electric fields in eq(21) are the ones generated at the rest frame of the axion stars. When the axion stars collide with neutron stars, the magnetic field $\vec{b} = \vec{v}_c \times \vec{E}_a$ with $v_c^2 \ll 1$ is induced at the rest frame of the neutron stars, where \vec{v}_c represents a relative velocity between the axion stars and the neutron stars. Since $v_c \sim 0.1$, the magnetic field \vec{b} is much smaller than \vec{B} . Thus, the effect can be neglected. Similarly, we can neglect the effects of the rotations of the neutron stars, whose velocities are much smaller than the relative velocities v_c .

As will be shown later, all the energy of a part of the axion star touching the atmospheres of neutron stars is released into the radiations, since the atmospheres of neutron stars are composed of highly dense electrons and ions. The electric fields of the axion stars induce oscillating electric currents which produce the radiations. Namely, the neutron stars make the axion stars evaporate into the radiations. The frequency of the radiations is given by $m_a/2\pi \simeq 2.4 \times (10^{-5}\text{eV}/m_a)\text{GHz}$ at the rest frame of the axion stars. Therefore, when the axion stars collide with neutron stars, the large amount of the radiations is produced within a short period R_a/v_c being of the order of milli seconds; v_c is of the order of 10^5km/s , see later. These radiations can escape the atmospheres and magnetosphere of neutron stars, because they are optically thin for the radiations as we show below. Thus, it is reasonable to identify the radiations as the FRBs observed.

IV. EVENT RATE OF FAST RADIO BURSTS

We calculate the rate of the collisions between axion stars and neutron stars in a galaxy. The collisions generate FRBs so that the rate is the event rate of the FRBs. By the comparison of theoretical with observed rate of the bursts, we can determine the mass of the axion stars. We assume that halo of a galaxy is composed of the axion stars whose velocities v relative to neutron stars is supposed to be $3 \times 10^2\text{km/s}$. Since the local density of the halo is supposed to be $0.5 \times 10^{-24}\text{g cm}^{-3}$, the number density n_a of the axion stars is given by $n_a = 0.5 \times 10^{-24}\text{g cm}^{-3}/M_a$. The event rate R_{burst} can be obtained in the following,

$$R_{\text{burst}} = n_a \times N_{\text{ns}} \times Sv \times 1\text{year}, \quad (23)$$

where N_{ns} represents the number of neutron stars in a galaxy; it is supposed to be 10^9 . The cross section S for the collision is given by $S = \pi(R_a + R_{\text{ns}})^2(1 + 2GM_{\text{ns}}/v^2(R_a + R_{\text{ns}})) \simeq 2.8\pi(R_a + R_{\text{ns}})GM_{\odot}/v^2$ where $R_{\text{ns}} (= 10\text{km})$ denotes the radius of neutron star with mass $M_{\text{ns}} = 1.4M_{\odot}$. It follows that the observed event rate is given by

$$\begin{aligned} R_{\text{burst}} &= \frac{0.5 \times 10^{-24}\text{g cm}^{-3}}{M_a} \times 10^9 \times 2.8\pi(10\text{km} + R_a) \frac{GM_{\odot}}{10^{-6}} \times 1\text{year} \\ &\sim 10^{-3} \left(\frac{10^{-12}M_{\odot}}{M_a} \right) \frac{10\text{km} + 260\text{km} \left(\frac{10^{-5}\text{eV}}{m_a} \right)^2 \frac{10^{-12}M_{\odot}}{M_a}}{10\text{km} + 260\text{km}}. \end{aligned} \quad (24)$$

Therefore, we can determine the masses M_a of the axion stars by the comparison of R_{burst} in eq(24) with the observed event rate $\sim 10^{-3}$ per year in a galaxy. We obtain $M_a \sim 10^{-12}M_{\odot}$ when $m_a = 10^{-5}\text{eV}$. The parameters used above still involves large ambiguities. Furthermore, the rate becomes larger than that in eq(24) when we take into account the cosmological evolution of the Universe. Thus, the observed rate only constrains the masses of the axion stars in a range such that $M_a = 10^{-12}M_{\odot} \sim 10^{-11}M_{\odot}$. It is remarkable that the mass $M_a \sim 10^{-12}M_{\odot}$ of the axion stars obtained is coincident with the masses of axion miniclusters[19] estimated previously.

Using the formula eq(18) we find the radius $\sim 10^2\text{km}$ of the axion stars with the mass $\sim 10^{-12}M_{\odot}$, which is larger than those of neutron stars. Then, when the collisions take place, the neutron stars pass through the insides of the axion stars. As we will show later, when the axion stars touch the atmospheres of the neutron stars, the large amount of the radiations is emitted instantaneously so that the parts of the stars touching them lose their energies. The amount of the radiation energy released in the collision is given by $10^{-12}M_{\odot}(10\text{km}/10^2\text{km})^2 \sim 10^{43}\text{GeV}$. Thus, our production mechanism of FRBs can explain the observed energies of the FRBs.

V. RADIATIONS FROM AXION STARS IN ATMOSPHERES OF NEUTRON STARS

We estimate how large amount of energies the axion stars emit as radiations in the collisions with neutron stars. In particular, we show that they rapidly lose their energies in the atmospheres[12] of neutron stars. We consider old neutron stars which are dominant components of neutron stars in the Universe. Their temperatures (magnetic fields) are assumed to be of the order of 10^5K (10^{10}G). We also assume that the neutron stars have hydrogen atmospheres.

First, we show how an electron emit radiations in the electric fields of the axion stars. The electric field \vec{E}_a on the axion stars generated under magnetic fields makes an electron oscillate according to the equation of motion $\vec{p} = (-e)\vec{E}_a + (-e)\vec{v} \times \vec{B} + m_e\vec{g}$ with electron mass m_e , where \vec{p} and \vec{v} denote momentum and velocity of the electron, respectively and \vec{g} does surface gravity of neutron stars; $|\vec{g}| = M_{ns}G/R_{ns}^2$. We note that the electric field \vec{E}_a is parallel to the magnetic field \vec{B} . Thus, the direction of the oscillation is parallel to \vec{B} . The magnetic field does not affect the oscillation. Similarly, the gravitational forces does not affect it since they are much weaker than the electric fields.

Then, the equation of motion of the electron parallel to \vec{E}_a is given by $\dot{p} = -eE_a$. Since the electric fields oscillate such as $E_a \propto \cos(m_a t)$, the electron oscillates with the frequency $m_a/2\pi$ so that it emits a dipole radiation. The amplitude of the oscillator is given by $e\alpha a_0 B/(m_a^2 m_e \pi) \simeq 0.05\text{cm}$ which is smaller than the wave length $\lambda \sim 10\text{cm}(10^{-5}\text{eV}/m_a)$ of the radiations. Thus, the emission rate of the radiation energy produced by a single electron with the mass m_e is given by

$$\dot{w} \equiv \frac{2e^2 \dot{p}^2}{3m_e^2} = \frac{2e^2(e\alpha a_0 B/\pi)^2}{3m_e^2} \simeq 0.7 \times 10^{-9} \text{GeV/s} \left(\frac{10^2 \text{km}}{R_a} \right)^4 \left(\frac{10^{-5} \text{eV}}{m_a} \right)^2 \left(\frac{B}{10^{10} \text{G}} \right)^2. \quad (25)$$

Electrons coherently oscillate in the volume λ^3 , in which there exist a number of the electrons with their number $N_e = n_e \lambda^3$ where n_e denotes the number density of electrons. Then, the total emission rate \dot{W} from the electron gas is given such that $\dot{W} = \dot{w}(n_e \lambda^3)^2 = 2(n_e \lambda^3)^2 \dot{p}^2/(3m_e^2)$. On the other hand, if the depth d of the atmosphere of neutron stars is less than the wave length of the radiations, the number of electrons coherently oscillating is given by $n_e d \lambda^2$. Actually, the depth d of the hydrogen atmosphere with temperature of the order of 10^5K is about 0.1cm , which is much smaller than the wave length $\sim 10\text{cm}(10^{-5}\text{eV}/m_a)$.

We make a comment that the thermal effects of electron gas under consideration do not disturb the oscillation by the electric field. Since the temperatures of the atmospheres are supposed to be 10^5K , the thermal energy $\sim 10\text{eV}$ of an electron is much smaller than the kinetic energy of the oscillation, $p^2/2m_e = (eE)^2/2m_e m_a^2 \sim 10^2 \text{eV}(B/10^{10}\text{G})^2$ with $m_a = 10^{-5}\text{eV}$. Although the oscillation is never disturbed in the thermal bath, the frequency of the radiations receives the effect of the thermal fluctuations so that the radiations have finite bandwidth.

We also make a comment about the depth d of atmospheres of neutron stars. We suppose that the density distribution $\rho(r)$ is given by $\rho(r) = \rho_0 \exp(-r/d)$ with the depth $d = k_B T/mg$ (m denotes average mass of the atoms composing the atmospheres, T does temperature of the atmospheres and k_B does Boltzmann constant.) The distribution may be obtained by solving the equation of the dynamical balance $\partial_r P(r) = -\rho(r)g$ between pressure P and surface gravity $g \equiv GM/R^2$ with the use of the equation of state $P(r) = n(r)k_B T$ of ideal gas. Here T denotes the constant temperature and M (R) and n denote mass (radius) of the star and number density ($n = \rho/m$) of atoms composing the atmosphere. For example, $d \sim 10\text{km}$ for $T = 300\text{K}$, $g_e = 9.8\text{m/s}^2$ and $m = 28\text{GeV}$ in the case of the earth, while $d \sim 0.1\text{cm}$ for $T = 10^5\text{K}$, $g_n = 10^{11} \times g_e$ and $m = 1\text{GeV}$ in the case of hydrogen atmosphere of neutron stars. Although the estimation is very rough, we can grip on the depth of the atmosphere of the neutron stars, which is given by 0.1cm when the temperature is of the order of 10^5K . We can see that the number density of electrons $n_e(r) = n_0 \exp(-r/0.1\text{cm})$ decreases rapidly with the distance r from the bottom of the atmospheres. The radiations emitted in the atmospheres can pass through the atmospheres without absorption because the atmosphere is transparent for the radiations with transverse polarizations, as shown in the next section.

We proceed to show that the axion stars rapidly evaporate into the radiations when they touch the atmospheres of neutron stars. We assume that the atmospheres are composed of fully ionized hydrogen gas with temperature of the order of 10^5K , whose depth d is about $\sim 0.1\text{cm}$. Thus, we have the density distribution $n_e(r) = n_0 \exp(-r/0.1\text{cm})$ where the density $n_e(r=0) = n_0$ at the bottom is much larger than $10^{24}/\text{cm}^3$. In the paper we take $n_0 = 10^{24}/\text{cm}^3$. It approximately corresponds to the density $1\text{g}/\text{cm}^3$. We consider the radiations arising from a region with volume $d\lambda^2 \sim 10\text{cm}^3$ in the atmospheres. The emission rate \dot{W} of the radiations from the region is given by,

$$\dot{W} \sim 10^{-9}(d\lambda^2 n_e)^2 \text{GeV/s} \left(\frac{B}{10^{10}\text{G}} \right)^2 \sim 10^{37} \text{GeV/s} \left(\frac{n_e}{10^{22}\text{cm}^{-3}} \right)^2 \left(\frac{10^2 \text{km}}{R_a} \right)^4 \left(\frac{10^{-5} \text{eV}}{m_a} \right)^6 \left(\frac{B}{10^{10}\text{G}} \right)^2, \quad (26)$$

where we have taken, for instance, the number density $n_e = 10^{22}/\text{cm}^3$ of electrons in the region with the density $\rho \sim 10^{-2}\text{g}/\text{cm}^3$, which is located roughly at the height $r = 0.5\text{cm}$. (As we take larger n_e , \dot{W} becomes larger.) On the

other hand, the energy of the axion stars contained in the volume $d\lambda^2 = 10\text{cm}^3$ is given by $10^{-12}M_\odot 10\text{cm}^3 / (4\pi R_a^3/3) \sim 10^{24}\text{GeV}$. This energy is smaller than the energy of the radiations $\dot{W} \times 10^{-11}\text{s} \simeq 10^{26}\text{GeV}$ emitted within a time $0.1\text{cm}/v_e \sim 10^{-11}\text{s}$ in which the axion stars pass the depth $d = 0.1\text{cm}$. It should be noted that the relative velocity v_e of the axion stars when they collide with the neutron stars, is given by $v_e = \sqrt{2G(1.4M_\odot)/R_{ns}} \simeq 6 \times 10^{-1} \simeq 2 \times 10^5\text{km/s}$. Therefore, we find that the whole energy of the region with the volume $\lambda^2 d$ in the axion stars is transformed into the radiation energy when the region pass through the atmospheres.

The purpose using the specific values $n_e = 10^{22}\text{cm}^{-3}$ or the depth $d = 0.1\text{cm}$ in the estimation is simply to show that the whole energies of the part of the axion stars passed through by neutron stars are transformed into radiations. Obviously, our results do not depend on the specific values. The use of different values similar to these ones does not change our results. Therefore, we conclude that the part of the axion stars touching the neutron stars are immediately evaporated into the radiations.

There are ambiguities about the parameters (temperature, density, composition, e.t.c) of neutron star atmospheres. Only what we need to derive our results is the fact that the average number density of electrons is much large such as $10^{22}/\text{cm}^3$ in the atmosphere and that strong magnetic fields $\geq 10^{10}\text{G}$ is present. These assumptions are generally acceptable. Thus, our production mechanism of the FRBs is fairly promising.

VI. TRANSPARENCY OF NEUTRON STAR ATMOSPHERE

The radiations produced in the atmospheres can pass through them and arrive at the earth. They are never absorbed within the atmospheres. We will show that the atmospheres are transparent for the radiations, even if $n_e \simeq 10^{22}\text{cm}^{-3}$ when the strong magnetic field stronger than $B = 10^{10}\text{G}$ is present. We assume that the temperatures of the atmospheres are of the order of 10^5K and that electron density is given by $n_e(r) = n_0 \exp(-r/0.1\text{cm})$, where the depth of the atmospheres is taken as 0.1cm , as was shown above. We also assume for simplicity that the atmospheres are composed of fully ionized hydrogen atoms.

Then, we may use the following formula[20] of the free-free absorption coefficient,

$$C_\epsilon(r) = \frac{n_e(r)}{(\omega + \epsilon\omega_c)^2 + \nu_\epsilon(r)^2} \frac{4\pi e^2 \nu_\epsilon(r)}{m_e} \quad (27)$$

with $\omega = m_a/2\pi$, $\omega_c = eB/m_e$ and $\omega_p = eB/m_p$, where m_p denotes proton mass and ν_ϵ is given by

$$\nu_\epsilon(r) = \frac{2e^2\omega^2}{3m_e} + \frac{4n_e(r)e^4\Lambda_\epsilon(T, B, \omega)}{3T} \sqrt{\frac{2\pi}{m_e T}} \quad (28)$$

where we used $\omega/T \ll 1$ since $T = 10^5\text{K}$ and $m_a = 10^{-5}\text{eV}$. The parameter $\epsilon = 0, \pm$ denotes three types of polarizations; circular polarizations $\epsilon = \pm$ (polarized transverse to \vec{B}) and longitudinal polarization $\epsilon = 0$ (polarized longitudinal to \vec{B}). The explicit formula of $\Lambda_\epsilon(T, B, \omega)$ is given by

$$\Lambda_\epsilon(T, B, \omega) = \frac{3}{4} \sum_{n=-\infty}^{\infty} \int_0^\infty Q_\epsilon(n, T, B, \omega, y) dy \quad (29)$$

where

$$Q_\epsilon(n, T, B, \omega, y) = \frac{y A_n^\epsilon(T, B, \omega, y)}{\sqrt{1 + 2\theta y + y^2} (y + \theta + \sqrt{1 + 2\theta y + y^2})^{|n|} (\sinh(b/2))^{|n|}} \quad (30)$$

$$A_n^0(T, B, \omega, y) = \frac{x_n K_1(x_n)}{y + b/4}, \quad A_n^\pm(T, B, \omega, y) = \frac{\omega^2}{(\omega \mp \omega_p)^2} \frac{(y + \theta + |n| \sqrt{1 + 2\theta y + y^2}) K_0(x_n)}{1 + 2\theta y + y^2} \quad (31)$$

$$b = 13.6 \frac{B}{10^{10}\text{G}} \frac{10^5\text{K}}{T}, \quad x_n = |\omega/T - nb| \sqrt{0.25 + y/b}, \quad \theta = \frac{1 + \exp(-b)}{1 + \exp(b)} \simeq 1 \quad (32)$$

with $x_0 \simeq |\omega/T| \sqrt{0.25 + y/b}$ and $x_{n \neq 0} = |nb| \sqrt{0.25 + y/b}$ since $\omega/T \simeq 10^{-7}$. K_0 and K_1 represent modified Bessel functions.

Here we note that the contributions of the sum over large integer n are very small because there are damping factors such as $1/(2^{|n|} \sinh(b/2)^{|n|}) \simeq 1/(800)^{|n|}$ and integrands of y have the factor $\exp(-|n|\sqrt{yb})$ for large y . The integration $\int_0^\infty dy \exp(-|n|\sqrt{yb})$ gives a damping factor n^{-2} for large n . Thus, the main contribution comes from the integral of $\int_0^\infty Q_\epsilon(n=0, T, B, \omega, y) dy$. We should also note the presence of the small factor $\omega^2/(\omega \pm \omega_p)^2 \simeq \omega^2/\omega_p^2 \simeq 10^{-7}(10^{10}\text{G/B})^2$ in A_n^\pm . The factor comes from the finiteness of proton mass; the recoil effect of the proton owing to the absorption of the radiations. Therefore, the absorption coefficient can be approximately rewritten by

$$C_\pm(r) \simeq \frac{n_e(r)}{\omega_c^2} \frac{4\pi e^2 \nu_\pm(r)}{m_e} \quad (33)$$

where

$$\nu_\pm(r) \simeq \frac{4n_e(r)e^4 \Lambda_\pm(T, B, \omega)}{3T} \sqrt{\frac{2\pi}{m_e T}} \simeq \frac{n_e(r)e^4 \int_0^\infty Q_\pm(n=0, T, B, \omega)}{T} \sqrt{\frac{2\pi}{m_e T}} \quad (34)$$

with $\theta \simeq 1$ and

$$Q_\pm(n=0, T, B, \omega) \simeq \frac{\omega^2}{\omega_p^2} \frac{y K_0(|\omega/T| \sqrt{0.25 + y/b})}{(1+y)^2}. \quad (35)$$

On the other hand, the absorption coefficient $C_0(r)$ for longitudinally polarized radiations is given by

$$C_0(r) \simeq \frac{n_e(r)}{\omega_c^2} \frac{4\pi e^2 \nu_0(r)}{m_e} \quad (36)$$

where

$$\nu_0(r) \simeq \frac{4n_e(r)e^4 \Lambda_0(T, B, \omega)}{3T} \sqrt{\frac{2\pi}{m_e T}} \simeq \frac{n_e(r)e^4 \int_0^\infty Q_0(n=0, T, B, \omega)}{T} \sqrt{\frac{2\pi}{m_e T}} \quad (37)$$

with

$$Q_0(n=0, T, B, \omega) \simeq \frac{y|\omega/T| \sqrt{0.25 + y/b} K_1(|\omega/T| \sqrt{0.25 + y/b})}{(1+y)(y+b/4)}. \quad (38)$$

Using these formulae, we can see the optical depth $\tau_\pm(r_c) = \int_{r=r_c}^\infty dr' C_\pm(r') < 1$ even at the location r_c in which the number density $n_e(r_c)$ is equal to $10^{22}/\text{cm}^3$. Therefore, we find that the atmospheres are transparent for the radiations with the circular polarizations. The transparency comes from the fact that the frequency $\omega = m_a/2\pi$ is much smaller than the cyclotron frequencies ω_c and ω_p under the strong magnetic fields $B = 10^{10}\text{G}$. Physically, the electric fields of the radiations hardly make electrons move transversely to the direction of the magnetic fields B . Thus, they cannot be absorbed. On the other hand, we can easily see that the atmospheres are opaque ($\int_{r=r_c}^\infty dr' C_0(r') \gg 1$) for the radiations with the longitudinal polarization since the radiations easily make electrons oscillate longitudinally; they are absorbed by the electrons.

We would like to mention that although the radiations emitted from the atmospheres are linearly polarized, some of them are circularly polarized after they pass through the magnetospheres of neutron stars. The magnetospheres are composed of electrons or positrons, which are produced by the Schwinger mechanism under electric field associated with the rotation of the magnetic field B . The charged particles are distributed to screen the electric field. The number density of electrons (positrons) in the magnetospheres is given by the Goldreich-Julian density $\simeq \Omega B/2\pi \sim 10^7 \text{cm}^{-3} (\Omega/(2\pi/\text{s})) (B(r)/10^{10}\text{G})$ with angular velocity Ω of neutron stars. These electrons (positrons) absorb right (left) handed circularly polarized radiations when the cyclotron frequency $\omega = eB(r)/m_e$ becomes equal to $m_a/2\pi$, respectively. Since $B(r)$ decreases such that $B(r) \propto 1/r^3$, the absorption arises around the location at the height $r_{ab} \sim 10^3 \text{km}$ above the surface of neutron stars. It implies that the absorption coefficient $C_\pm(r_{ab})$ is much large for a type of circularly polarized radiations compared with the one for the other type of circularly polarized radiations, for instance, $C_+(r_{ab}) \gg C_-(r_{ab})$. The spatial distribution of the electrons is different from the distribution of the

positrons. Therefore, the radiations passing through the magnetospheres are circularly polarized. Such a polarization has been observed[4] in FRB 140514.

We would like to point out that the atmospheres may evaporate instantaneously when the radiations pass them. This is because even if only a fraction of the radiation energies is dissipated in the atmospheres, the energy is sufficiently large to make the atmospheres evaporate. For example, a fraction e.g. 10^{-8} of the radiation energies 10^{43}GeV , ($10^{-8} \times 10^{43}\text{GeV} = 10^{35}\text{GeV}$) gives a large energy $10^{35}/(N = 10^{35}) = 1\text{GeV}$ to each nucleon in the atmospheres; $N \sim 10^{24}/\text{cm}^3 \times 0.1\text{cm}(10^6\text{cm})^2 = 10^{35}$. Then, it apparently seems that our production mechanism of the FRBs does not work. But we should note that the FRBs are also produced in envelopes present just below the atmospheres. The envelopes are more dense ($10^{24}/\text{cm}^3 \sim 10^{32}/\text{cm}^3$) in electron number density and deeper ($\sim 10^4\text{cm}$) than the atmospheres. Thus, even if the atmospheres instantaneously evaporate, the radio bursts with sufficiently large energies as observed are produced in the envelopes of neutron stars.

VII. NARROW BANDWIDTH

It apparently seems that the radiations are monochromatic, that is, their frequencies are given by the axion mass. On the other hand, the FRBs have been observed with the frequencies in the range of $1.2\text{GHz} \sim 1.6\text{GHz}$. Here we would like to show that the radiations emitted from the neutron stars have finite bandwidth including the range of the observed frequencies. They are dipole radiations emitted by electrons harmonically oscillating. These electrons have temperatures of the order of $10^5\text{K} \simeq 10\text{eV}$. Thus, we take account of thermal effects on the oscillations. The kinetic energies ϵ_k of the oscillations are given by $p^2/2m_e = (eE)^2/2m_em_a^2 \sim 10^2\text{eV}(B/10^{10}\text{G})^2$ with $m_a = 10^{-5}\text{eV}$. The energy is equal to the potential energy $m_e\omega^2 x_e^2/2$ of the harmonic oscillations with the frequency $\omega = m_a/2\pi$; x_e represents the amplitude of electrons. When the thermal fluctuations are added to the harmonic oscillations, the electron motion may be described by the following Langevin equation,

$$m_e\ddot{x}_e = -m_e\omega^2 x_e + \eta \quad (39)$$

where the thermal fluctuation is represented by η . Since we consider only the effect on the harmonic oscillation $x_e = x_0 \cos(\omega t)$, we take only the term of $\eta = \eta_0 \cos(\omega' t)$. Then, the frequency $\omega' = \omega + \omega_{\text{th}}$ ($\omega \gg \omega_{\text{th}}$) of electrons can be derived from the Langevin equation such that $m_e(\omega'^2 - \omega^2)x_0 \simeq 2m_e\omega\omega_{\text{th}}x_0 = \eta_0$; $\omega_{\text{th}} = \eta_0/(2m_e\omega x_0)$. Thus, the fluctuations ω_{th} in the frequency is obtained by taking average of the thermal fluctuation η_0 with an appropriate Gaussian distribution,

$$\langle \omega_{\text{th}}^2 \rangle = \frac{1}{4m_e^2\omega^2 x_0^2} \langle \eta_0^2 \rangle = \frac{\omega^2 \langle y^2 \rangle}{4x_0^2} = \frac{T}{2m_e x_0^2} = \frac{\omega^2 T}{4\epsilon_k}, \quad (40)$$

with $\eta_0 \equiv m_e\omega^2 y$. The Gaussian distribution is assumed to be given by $\exp(-m_e\omega^2 y^2/4T)$.

Thus, the thermal fluctuations in the frequencies of electrons are given by

$$\omega \pm \omega_{\text{th}} = \omega(1 \pm \frac{\omega_{\text{th}}}{\omega}) = \frac{m_a}{2\pi} \left(1 \pm \sqrt{\frac{T}{4\epsilon_k}}\right) \simeq \frac{m_a}{2\pi} \left(1 \pm \sqrt{\frac{10\text{eV}}{4 \times 10^2\text{eV}}}\right) \simeq \frac{m_a}{2\pi} (1 \pm 0.16), \quad (41)$$

where we take values $T = 10^5$, $B = 10^{10}\text{G}$ and $m_a = 10^{-5}\text{eV}$. The thermal fluctuations of the electrons cause the finite but narrow bandwidth of the FRBs. We should note that the fluctuation $\omega_{\text{th}}/\omega$ depends on the temperature T , magnetic field B and axion mass m_a ,

$$\frac{\omega_{\text{th}}}{\omega} = \sqrt{\frac{T}{4\epsilon_k}} \sim 0.1 \frac{m_a}{10^{-5}\text{eV}} \frac{10^{10}\text{G}}{B} \sqrt{\frac{T}{4\text{eV}}}. \quad (42)$$

The equation is used for the estimation of the bandwidths of the radio bursts from the collision between axion stars and white dwarfs, which is discussed in next section.

We should mention that the observed radiations receive several redshifts. The frequency of the electric fields induced on the axion stars under the magnetic fields is equal to $\omega = m_a/2\pi$. Since the axion stars collide with the neutron stars at the relative velocity $\sqrt{2GM_{ns}/R_{ns}}$, the frequencies ω_{ns} of oscillating electrons induced by the electric fields is given by $\omega_{ns} = \omega\sqrt{1 - 2GM_{ns}/R_{ns}}$ at the rest frame of the neutron stars. Thus, the radiations with the frequency

ω_{ns} are emitted by the electrons at the rest frame of the neutron stars. The radiations receive gravitational redshifts when we observe them far from the neutron stars. The frequency ω' is given by $\omega' = \omega_{ns} \sqrt{1 - 2GM_{ns}/R_{ns}}$. Finally, the frequency of the radiations observed at the earth is given by $\omega_{ob} = \omega'/(1+z) = \omega(1 - 2GM_{ns}/R_{ns})/(1+z)$ when the neutron stars are located at the places with redshift z .

VIII. COLLISIONS WITH MAGNETIC WHITE DWARFS

Up to now, we have considered that FRBs arise from the collisions between axion stars and neutron stars. Similarly, FRBs may arise from the collisions between axion stars and magnetic white dwarfs[21]. Some of the magnetic white dwarfs have strong magnetic fields such as 10^9G . They have dense hydrogen atmospheres with temperatures of order of 10^4K and depths of the order of 10^4cm . (We can easily estimate the depth by taking account of the physical parameters, the surface gravity $g_{wd} \simeq 10^5 \times g_e$ and the temperature $T = 10^4\text{K}$ of the white dwarfs. Thus, the density distribution is given by $n = n_0 \exp(-r/10^4\text{cm})$.) They have dense free electrons similar to the case of neutron stars. Thus, by the collisions with axion stars, radiation bursts similar to the observed FRBs are produced. It turns out that the duration of the bursts is of the order of 0.1 second. This is because the axion stars collide with white dwarfs at the velocity $\sqrt{2GM_{wd}/R_{wd}} \simeq 4000\text{km/s}$ where the mass M_{wd} and radius R_{wd} of the white dwarfs are typically given by $0.5M_\odot$ and 10^4km , respectively. Thus it approximately takes 0.1 second for the axion stars to pass the atmospheres of the white dwarfs. A distinctive feature is that the radiations from the magnetic white dwarfs with $B \sim 10^9\text{G}$ have wider bandwidths than those of the radiations from the neutron stars. Since the temperatures of the white dwarfs are equal to or less than 10^4K and the magnetic field is equal to 10^9G , we find using eq(42) that the fluctuations ω_{th}/ω is three times larger than those of the radiations from the neutron stars with $T = 10^5\text{K}$ and $B = 10^{10}\text{G}$.

When we observe them at earth, they receive gravitational and cosmological red shifts. The effect of the gravitational red shift is, however, very small; $\sqrt{1 - 2GM_{wd}/R_{wd}} \simeq 1$. Hence, the frequencies observed at the earth are given by $(\omega \pm \omega_{th})(1+z)^{-1}$, which are larger than the frequencies of the radiations from neutron stars located at the places with the redshift z .

We should mention that the radio bursts from white dwarfs with $B = 10^9\text{G}$ are more energetic than those of the observed FRBs. This is because the whole energies of the axion stars colliding with such white dwarfs are transformed into radiations. The energies are of the order of $10^{-12}M_\odot \simeq 10^{43}\text{erg}$. As we have shown, the axion stars are much smaller than the white dwarfs so that the whole of axion stars collide with the white dwarfs.

Actually, the emission rate of the radiation energy produced by a single electron in the atmospheres is given by

$$\dot{w} \equiv \frac{2e^2\dot{p}^2}{3m_e^2} \simeq 0.7 \times 10^{-11}\text{GeV/s} \left(\frac{10^2\text{km}}{R_a}\right)^4 \left(\frac{10^{-5}\text{eV}}{m_a}\right)^2 \left(\frac{B}{10^9\text{G}}\right)^2, \quad (43)$$

where the radiations are also dipole ones.

The electrons in a volume λ^3 ($\lambda = 2\pi/m_a \simeq 10\text{cm}(10^{-5}\text{eV}/m_a)$ denotes the wave length of the radiations) coherently emit the radiations. Thus, the emission rate of the coherent radiations in the volume is

$$\dot{w}(n_e\lambda^3)^2 \simeq 10^{39}\text{GeV/s} \left(\frac{B}{10^9\text{G}}\right)^2 \left(\frac{n_e}{10^{22}\text{cm}^{-3}}\right)^2 \quad (44)$$

where n_e denotes number density of electrons in the atmospheres. On the other hand, the axion stars have the energies $10^{-12}M_\odot(\lambda/R_a)^3 \simeq 10^{33}\text{GeV}$ in the volume λ^3 . Thus, when the axion stars pass the atmospheres in a period 0.1second, their whole energies are transformed into the radiations. In this way, when the axion stars collide with the magnetic white dwarfs, they disappear emitting the radiations.

It is easy to show that the atmospheres of the white dwarfs are transparent for the radiations. In the formulae given above, taking the parameters $B = 10^9\text{G}$ and $T = 10^4\text{K}$, we find that the optical depth $\tau_\pm(r_c) = \int_{r=r_c}^\infty dr' C_\pm(r') < 1$ even at $r = r_c$ in which $n(r_c) = 10^{22}\text{cm}^{-3}$.

We should make a comment that the typical white dwarfs have magnetic fields $B \sim 10^7\text{G}$ much smaller than 10^9G . It leads to the numerical parameters $\omega^2/\omega_p^2 \simeq 10^{-3}$, $b \simeq 0.134$ and $\theta \simeq 1/b$. Then, we find that the atmospheres are not transparent when the radiations are produced in the deep inside of the atmosphere with the electron density such as $n_e = 10^{22}\text{cm}^{-3}$. However, when they are produced at the depth with $n_e = 10^{13}\text{cm}^{-3}$, they can pass through the atmospheres. But the emission rate in the volume λ^3 is much small,

$$\dot{w}(n_e\lambda^3)^2 \simeq 10^{17}\text{GeV/s} \left(\frac{B}{10^7\text{G}}\right)^2 \left(\frac{n_e}{10^{13}\text{cm}^{-3}}\right)^2. \quad (45)$$

Thus, the axion stars emit radiations with their energies $\dot{w}(n_e\lambda^3)^2(R_a/\lambda)^3 \sim 10^{35}\text{GeV/s}$. Hence, the collisions between the white dwarfs with $B \sim 10^7\text{G}$ and the axion stars does not produce the radiations with enough luminosities to be observed at the earth when they arise in extragalactic origins.

If the number of white dwarfs in a typical galaxy is of the order of 10^{12} , only a small fraction $10^{-4} \sim 10^{-5}$ of the white dwarfs would be those with strong magnetic fields $\geq 10^9\text{G}$ and hydrogen atmospheres. Then, the production rate R_{burst} of the FRBs emitted in the collisions with such magnetic white dwarfs is found such that $R_{burst} \sim (10^{-2}/\text{year} \sim 10^{-3}/\text{year})$ in a galaxy. The values are obtained by using the formula in eq(24) with the use of $R_{wd} = 10^4\text{km}$. The rate is ten times larger than or equal to the rate of the FRBs actually observed. In other words, if the typical number of such magnetic white dwarfs in a galaxy is of the order of 10^7 , the rate of the bursts is approximately equal to the rate of the FRBs observed. Thus, the FRBs associated with the white dwarfs can be observed with their frequencies in a range $2\text{GHz} \sim 3\text{GHz}$. Obviously, they can be distinguished from those arising from the collisions with neutron stars.

The number of the white dwarfs with strong magnetic fields $\geq 10^9\text{G}$ in a galaxy is not known and the estimation of the number is difficult. Although we know the presence of such white dwarfs, the number of them could be very few. Thus, the event rate of the FRBs associated with the white dwarfs[21] could be much small so that the FRBs are undetectable.

IX. SUMMARY AND DISCUSSIONS

We have shown the details of a possible production mechanism of FRBs; FRBs arise from the collisions between axion stars and neutron stars. We have found that the masses and radii of the axion stars are given by $M_a \sim 10^{-12}M_\odot$ and $R_a \sim 10^2\text{km}$, respectively. The axion stars are rapidly converted into radiations under strong magnetic fields of the neutron stars. The radiations are emitted in the atmospheres of the neutron stars. We have shown that the atmospheres are transparent for the radiations. The transparency comes from the presence of strong magnetic fields $B \geq 10^{10}\text{G}$ and the low frequencies $\sim 1\text{GHz}$ of the bursts. According to the mechanism, we can explain naturally the durations ($\sim \text{ms}$) and amount of the energies (10^{40}erg) of the bursts.

It apparently seems that the radiations is monochromatic with the frequency given by the axion mass. But, the observed frequencies have finite bandwidths. We have shown that the bandwidths are caused by the thermal fluctuations of electrons emitting the radiations.

In the actual collisions the tidal forces of the neutron stars distort the formation of the axion stars. When the axion stars are close to the neutron stars, the gravitational forces of the neutron stars are stronger than those of axion stars binding themselves. Then, the axions freely fall to the neutron stars. But the coherence of the axions is kept because the number density of the axions in the volume m_a^{-3} is quite large.

Our mechanism predicts that there are no radiations with any frequencies after the bursts. This is consistent with the results of follow-up observations[4]. It also predicts that FRBs contain circular polarizations. The circular polarizations arise owing to the absorption of either right or left handed polarized radiations in the magnetospheres. Circular polarizations have recently been observed[4] in a FRB.

Similar radio bursts may arise when the axion stars collide with magnetic white dwarfs. We have found that the duration of the bursts is of the order of 0.1second and that the radiations have wider bandwidths than those of the radiations from neutron stars. The features can be observable only if the white dwarfs have strong magnetic fields $\geq 10^9\text{G}$. Although the number of such white dwarfs in a galaxy is unknown, the production rate of the bursts is sufficiently large for them to be detectable if their number is larger than 10^6 in a galaxy.

If the our production mechanism of FRBs is true, we can reach a significant conclusion that the axions are the dominant component of dark matter and their mass is about 10^{-5}eV , which is in the window allowed by observational and cosmological constraints[22].

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